Some Motivating Questions

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Time reversibility, linear models, growth collapse prediction

Linear least squares is time reversible - will not differentiate between streamflow and price of silver.

Non-linear L.S. will give better prediction for slow-growth fast-collapse than for fast growth slow collapse, but is it good?

Maximum likelihood will possibly ignore impeding collapse or next impulse.

Should we use bimodal prediction intervals?

Price of silver,
(J. Michael Steele, Financial time series - honest data analysis)
Slow growth, and collapse
**Bandits, restless bandits, discounted restless bandits**

Worked with Isaco Meilijson, with Esti Frostig, and Richard Weber.

New book:
**Multi-armed Bandit Allocation Indices, 2nd Edition**
*John Gittins, Kevin Glazebrook, Richard Weber*

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**Stochastic scheduling: How bad is worst case?**

(Many years of work with Mike Pinedo, Rolf Moehring, Rhonda Righter etc.)

Worst case analysis of heuristics is terribly depressing, the most plausible heuristics perform terribly if you challenge them.

But is it really so bad when the problem data is stochastic?

**Example:**

Minimizing weighted sum of completion times on parallel machines:

- **Smith Rule** or "$c^\mu$" rule optimal on single machine
- For two machines: NP-hard.

Smith Rule worst case performance ratio:

$$\sum_{i=1}^{n} \frac{W_i C_i}{\text{Opt}} \leq 1.207$$

Do we really lose 20% efficiency?

Only by constructing a very contrived example!

An "average problem" will be much better behaved but what is average?

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**Cafeteria, Layers of Poisson process, Hammerseley’s I.P.S.:**

Cafeteria, with Richard Weber

Layers of the Poisson process in the plane, with Tomasz Rolski and the late Benny Levikson

Allows us to calculate moments of the distribution of the longest increasing subsequence in a permutation (Hammerseley’s problem), same idea used by Aldous Diaconis to define an I.P.S.

What does it say about the output of an infinite sequence of queues with blocking?
Control of Processing Networks

The stochastic idea: Model deviations from fluid solution as piecewise homogeneous queueing network with infinite virtual queues

Working with $p=1$: Max-Pressure is rate stable
Tracking asymptotically optimal,
Diffusion approximation may show $N^{1/2}$ tracking deviation

with Nazarathy, Lefeber, Zhang, Guo we believe:
If each resource has IVQ,
exist policies that are positive Harris recurrent
Tracking with $O(\text{constant})$ deviation

with Jim Dai: For a network with a single bottleneck,
Safety stock of $\log(N)$ is sufficient for minimal makespan

Q: Can we track finite horizon of $N$ items within $\log(N)$ of fluid solution?

Continuous LPs and Dynamic Linear Complementarity
(with John Hasenbein, Leon Lasdon, Ron Billings & Evgeny Shindin)

$$\begin{align*}
\max & \ c' u \\
\text{s.t.} & \ b' v \\
& \ u \geq 0, \ v \geq 0
\end{align*}$$

CLP

$$\begin{align*}
\max & \ int_{0}^{T} c(T-t) dU(t) \\
\min & \ int_{0}^{T} b(T-t) dV(t) \\
AU(t) & \leq b(t) \\
U(t) & \nearrow \\
AV(t) & \geq c(t) \\
V(t) & \nearrow
\end{align*}$$

Solving by parametric Lemke algorithm:
Continuous LPs and Dynamic Linear Complementarity

\[ \text{LCP}(q,M) : \quad z = q + Mw, \quad z \geq 0, \quad w \geq 0, \quad z^t w = 0 \]

\[ \phi(x) = x^+ - M x^-, \quad z = x^+ w = x^- \] solves \ LCP(\phi(x),M) \quad x = \phi^{-1}(q) \]

\[ \phi : \mathbb{R}^n \rightarrow \mathbb{R}^n \text{ c.p.w.} \]

Linear transformation of orthants to convex polyhedral cones,

Lemke's Algorithm

| \begin{align*}
q & = \begin{bmatrix} b \end{bmatrix} \\
L & = \begin{bmatrix} -A & A' \end{bmatrix} \\
w & = \begin{bmatrix} v \\ u \end{bmatrix} \\
z & = \begin{bmatrix} b - Au \\ A'v - c \end{bmatrix}
\end{align*} |

Convex Quadratic Programming,

Especially 2-person bi-matrix games:

Lemke's algorithm finds a Nash equilibrium

Binatrix Game:

\[ q = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \]

\[ M = \begin{bmatrix} 0 & A' \\ B' & 0 \end{bmatrix} \]

\[ w = \begin{bmatrix} v \\ u \end{bmatrix} \]

\[ z = \begin{bmatrix} Av - 1 \\ B'v - 1 \end{bmatrix} \]

Questions: Does it work for Continuous Convex Quadratic Programming?

What does it solve?

Does it work for Continuous Bi-matrix Games?

What does it solve?

What is the interpretation of these "Continuous Bi-matrix Games"?
Multi-type server & queues - Many server scaling under FCFS

Time reversal proof -
(Kelly page 28)

Product form solution

Max Flow Decomposition

ED - Efficiency Driven

QD - Quality Driven